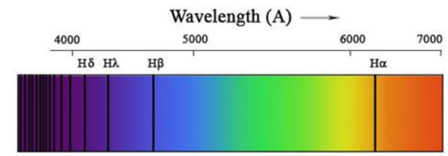
I. Key Experiments and their implications

- You learned a large part of these key experiments in PHYS1122
- Review coverage in PHYS1122 for details
- Here, we focus on the main implications on what quantum mechanics needs to handle and to explain
- Another interesting aspect is the big group of extraordinary physicists whose works contributed to formulating QM
  - Classical Physics: Lagrange, Hamilton, Jacobi
  - Quantum Physics: Balmer, Rydberg, Röntgen, J.J. Thomson, M. Curie, Zeeman, Planck, Einstein (1905, 1908, 1917, 1924), Millikan, Rutherford and Marsden, Bohr, de Broglie, Stern, Gerlach, Goudsmit, Uhlenbeck, Pauli, Heisenberg, Born, Jordan, Schrödinger, Dirac, Fermi, Bragg, Davisson, Franck, Compton, G.P. Thomson, and many more

A. Big Problems for classical physics (~1900)

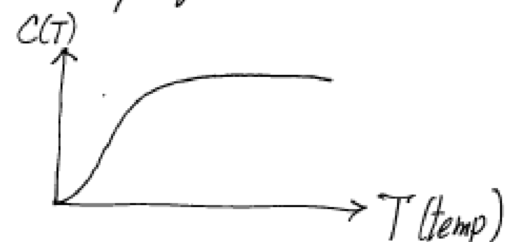
- Existence of atoms [Why not  ?]



- Spectrum characteristic of each atom [H, He, Li, ∘∘∘, each has its own spectrum]
- Thermal (black-body) radiation

How come?

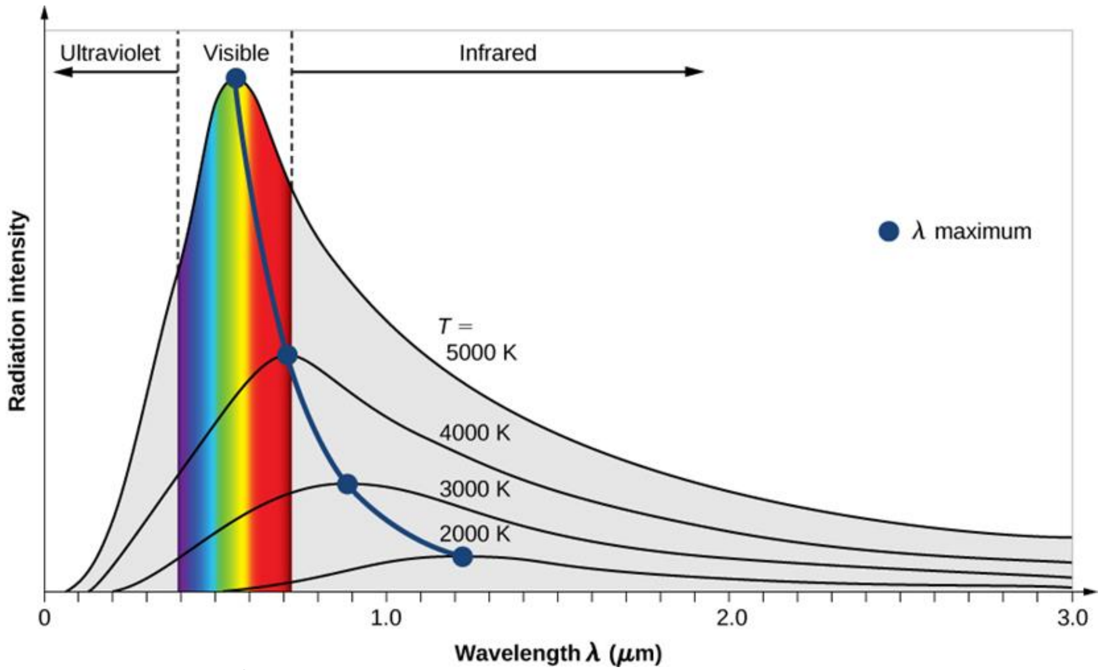
- Photoelectric effect [e's come out only for $\nu >$ threshold]
- Heat capacity of solids



- In studying these problems, a deeper understanding of ...
  - light [particle nature of light]
  - particle [wave nature of particle]

emerged

B. Thermal Radiation†



- Note : x-axis is wavelength λ
- Exp'ts done by Lummer, Pringsheim, Rubens, Kurlbaum [unknown heros] around 1900 in Berlin
- $T \lambda_{max} = \text{constant} = 0.0029\, m \cdot K$ [Wien's law]
- e_{tot} = energy per volume per second (all wavelengths) $= \sigma \cdot T^4$ [Stefan-Boltzmann law]

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† This is usually called "Black-body" radiation, but we don't want to go into a detailed discussion on what "black-body" really means. It is largely unnecessary. The point is every object at equilibrium at a finite temperature emits EM radiation. When the spectrum depends only on T but not the material, it is "black-body" radiation.

- <u>What classical physics and mathematics can do?</u>

Mathematics: $\mathcal{E}_{tot} \sim T^4$ (adding up contributions from all λ's)

Let $u(\lambda,T)\,d\lambda$ = energy per volume from contributions in the range of wavelengths from λ to $\lambda + d\lambda$

Pay attention here

You will encounter many quantities defined analogously in physics [e.g. $P(x)dx$ = Prob. of finding a particle at positions in the range from x to $x+dx$]

Reason: quantity concerned is <u>continuous</u>

Back to thermal radiation: $u(\lambda,T)\,d\lambda \sim \frac{1}{\lambda^5}\varphi(\lambda T)\,d\lambda$ to satisfy $\sim T^4$ law

Wien [1911 Nobel Prize]

Planck got the correct form [1918 Nobel Prize] Why? (Ex.)

$\boxed{\text{unknown function}}$

(λT) is the variable

Classical Physics: $u(\lambda, T)\,d\lambda \sim \underbrace{\frac{T}{\lambda^4}}\,d\lambda$ [Jean and Rayleigh]

[classical statistical physics]

Key point: It doesn't work!

- Only works for long wavelengths
- Unphysical behavior for short wavelengths
- e_{tot} diverges! [UV catastrophe]

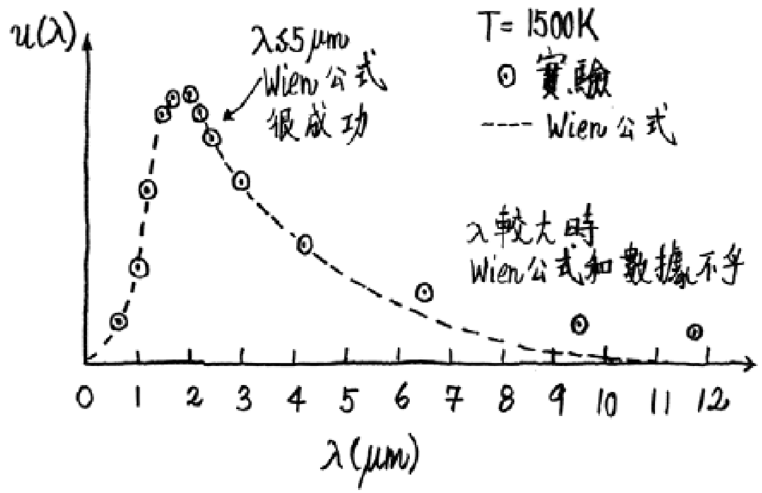
- Wien[+] guessed a form [worked only approximately]

He said: $\frac{b}{\lambda^5}\underbrace{e^{-a/\lambda T}}$ [a, b : fitting parameters]

- saved the problem in short wavelengths
- but doesn't work well over whole range of λ

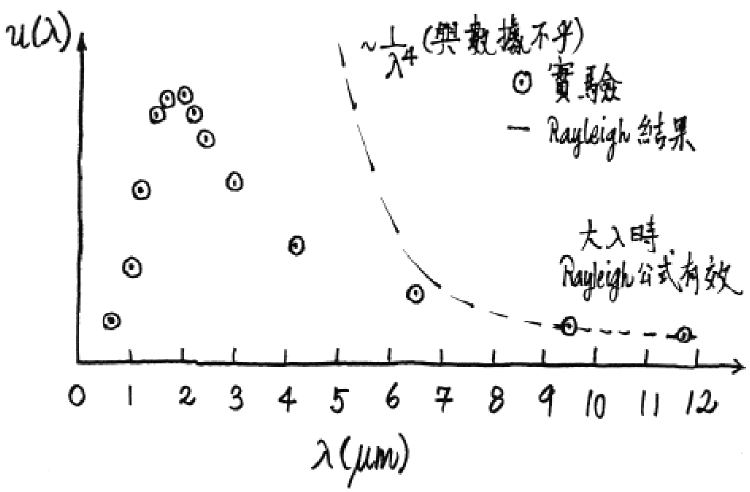
[1911 Nobel Prize]

[+] Years later, Wien wanted to fail Heisenberg after attending his PhD oral defense and Sommerfeld came to his rescue.



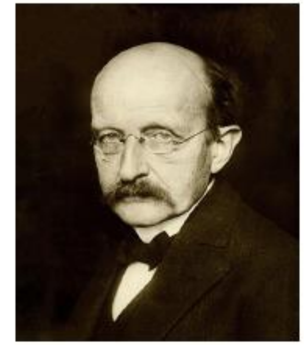
Wien's formula worked quite well until ~1900 when measurements could be done at longer wavelengths – this is how science develops

Wien's formula *only* works for short wavelengths (high frequencies)



Jean-Rayleigh classical physics approach *only* works for long wavelengths (low frequencies)

Implication: The correct formula should give these two limits and connect them



Max Planck
(1858-1947)

- Here came Planck
  - What to do when there was no theory?

"Fit" a curve by inspection & by insight!

Planck suggested a form: $\underbrace{\dfrac{C_1}{\lambda^5}\dfrac{1}{e^{C_2/\lambda T}-1}}$ (C_1, C_2 are fitting parameters)

(no theory)

(1900)

Note: reduced to Wien's form for short wavelengths and to $\sim\frac{T}{\lambda^4}$ for long wavelengths

[A clever form that is known to work in two limits]

$$\boxed{u(\lambda,T)\,d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}}-1}\,d\lambda}^{+}$$ works perfectly!

+ In PHYS1122, you saw an expression for the spectral distribution of radiation

$$I(\lambda)\,d\lambda = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}}-1}\,d\lambda = \frac{c}{4}\cdot\underbrace{u(\lambda,T)}_{\text{our discussion focuses on } u(\lambda,T)}d\lambda$$

- A historical moment!
  - There is c (speed of light) [EM radiation]
  - There is k_B (Boltzmann's constant) [thermal/statistical physics, temperature T]
  - Here enters h (Planck's constant $\underbrace{h = 6.626 \times 10^{-34}\ J \cdot s}_{\text{smallness} \Rightarrow \text{quantum effects are not daily life experience}}$)

signature of something quantum in the phenomenon

- The Planck formula works so well that it is used in
  - design of thermometers (thermometry)
  - CMB studies (Cosmic Microwave background)
  - identifying the effects of green house gases in atmosphere
- Accurate measurements of h ⇒ re-defining kilogram (possibly 2017)

- <u>Implication</u> : <u>Entering the Quantum Era</u>

EM: $\boxed{c\underset{\text{wave number}}{k} = \underset{\text{angular frequency}}{\omega}}$ OR $\boxed{c\cdot\frac{2\pi}{\lambda} = 2\pi f}$ OR $\boxed{c = \lambda\cdot f}$ OR $\boxed{\frac{c\cdot 2\pi}{\lambda} = \omega}$

- $u(\lambda,T)d\lambda$ concerns the range of <u>wavelengths</u> λ to $\lambda + d\lambda$

- Can also consider the range of <u>frequences</u> f to $f+df$ [+]
- Can also consider the range of <u>angular frequencies</u> ω to $\omega + d\omega$

<u>Answers are</u>:

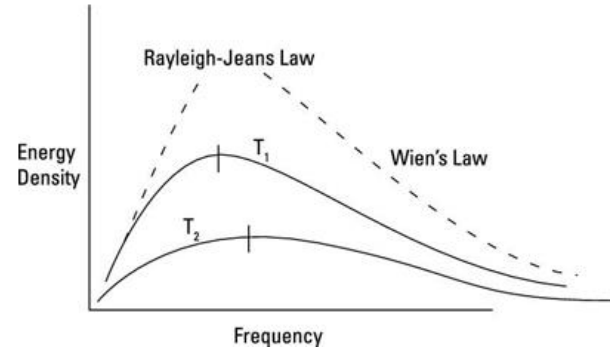
$$\boxed{u(f,T)df = \frac{8\pi f^2}{c^3}\,\frac{hf}{e^{\frac{hf}{k_BT}}-1}\,df} \quad \& \quad \boxed{u(\omega,T)d\omega = \frac{\omega^2}{\pi^2c^3}\,\frac{\hbar\omega}{e^{\frac{\hbar\omega}{k_BT}}-1}\,d\omega}$$

Planck saw <u>a quantum implication</u> in his formula.

[+] Try to transform Planck's expression from λ to f and ω. This is a good test on your math skill. Don't feel bad if you can't get it right in one attempt.

Let's consider: $u(f,T)\,df = \underbrace{\left(\frac{8\pi f^2}{c^3}\right)}_{\boxed{\text{• 3D • } c=\lambda f \text{ give this term}}} \cdot \underbrace{\left(\frac{hf}{e^{\frac{hf}{k_BT}}-1}\right)}_{\text{• } h \text{ appears here • focus on this term}} df$

Nothing quantum → (not our focus)



- <u>Physical Picture</u>
  - Atoms in walls of cavity (Matter) are coupled (bonded)
    ⇒ they oscillate over a range of frequencies
    [many oscillators each with a characteristic freq. f]
  - These oscillators can be excited and de-excited when they absorb or emit light (in the cavity)
  - Key point: $\underbrace{\text{Matter-Light interaction}}$, helps achieve equilibrium

<u>classical physics</u>: each oscillator (any f) carries k_BT energy

same as $\frac{T}{\lambda^4}$

if so, $u(f,T)\,df = \frac{8\pi}{c^3} f^2 \cdot k_BT\, df$ [Jean & Rayleigh] (doesn't work)

- Planck [Annalen der Physik (1901)] saw that his formula would imply
  - $\boxed{\text{energy could not be absorbed or emitted in any arbitrary amount}}$

    this will lead to kT per oscillator
  - instead Planck suggested for an oscillator of frequency f, its energy is not "a continuous, infinitely divisible quantity, but a discrete quantity composed of an integral number of finite equal parts"

Planck: $E_{\substack{\text{oscillator}\\ \text{of freq. } f}} = 0, hf, 2hf, 3hf, \ldots$

$\Rightarrow$ an oscillator can emit or absorb energy (radiation) only in small "packets" called $\underbrace{\text{Quanta}}_{\text{plural of quantum}}$ given by $\underbrace{E_{\text{Quantum}} = hf}_{\substack{\text{light is absorbed \& emitted}\\ \text{in such light quantum}\\ \text{[earliest mention of photons]}}}$

(energy quanta)

How come?

Planck used statistical physics (Boltzmann ~1880)

- for a system at equilibrium at temp. T, it has an energy E with probability $\propto e^{-E/k_BT}$

Now, for an oscillator of frequency f:

energy	probability
0	$\sim e^{-0/k_BT}$
hf	$\sim e^{-hf/k_BT}$
$2hf$	$\sim e^{-2hf/k_BT}$
$\vdots$	$\vdots$
nhf	$\sim e^{-nhf/k_BT}$
$\vdots$	$\vdots$

Averaged energy of oscillator

$$= \frac{0\cdot e^{-\frac{0}{k_BT}} + hf e^{-\frac{hf}{k_BT}} + \cdots + nhf e^{-\frac{nhf}{k_BT}} + \cdots}{e^{-\frac{0}{k_BT}} + e^{-\frac{hf}{k_BT}} + \cdots + e^{-\frac{nhf}{k_BT}} + \cdots}$$

$$= \frac{\sum_{n=0}^{\infty} nhf\, e^{-\frac{nhf}{k_BT}}}{\sum_{n=0}^{\infty} e^{-\frac{nhf}{k_BT}}} \qquad \left(\beta \equiv \frac{1}{k_BT}\right)$$

$$= -\frac{\partial}{\partial\beta}\left[\ln\left(\sum_{n=0}^{\infty} e^{-nhf\beta}\right)\right] \qquad (\text{Ex.})$$

$$= -\frac{\partial}{\partial\beta}\left[\ln\left(\frac{1}{1-e^{-hf\beta}}\right)\right] \qquad (\text{Ex.})$$

$$= \frac{hf}{e^{hf/k_BT}-1} \qquad [\text{factor in Planck's formula!}]$$

The Physics:

- No more UV catastrophe at high frequencies

- $hf \gg k_B T$, thermal energy $k_B T$ is not sufficient to excite the oscillator

- Thus, no more excitation and de-excitation at such frequencies f and that's why the radiation curve drops at high frequencies

- Energy Quanta (hf) set an energy scale to compete with $k_B T$

- It is quantum physics and statistical physics in action

Summary

- Exp't $\Rightarrow$ Planck's formula and introduced h
- Planck's formula asks for:
  - allowed energies of an oscillator are: $0, hf, 2hf, 3hf, \ldots$

    [Meaning: Quantum Mechanics applied to oscillator must give these discrete values]

    (This part is about matter)
- Planck's formula hints at:
  - Light is absorbed and emitted in packets of hf (hence hinted at photons)

Appreciation

- A great experiment and a great example of the nature of science
- h is well and alive! It will re-define the kilogram!
- 1918 Nobel Prize citation: "... to the advancement of Physics by his discovery of energy quanta"

Later developments:

Planck's formula led to ground-breaking developments every time it was re-visited

Einstein (1917) – Realized Planck's formula requires a (then) new phenomenon of ***Stimulated Emissions*** (how lasers work)

Bose (1924) – Realized Planck's formula can be treated as a statistical mechanics problem of ***a gas of photons***

Einstein (1924) – After reading Bose's manuscript, realized that Bose's method also worked for real matter (e.g. atoms) in addition to photons (non-matter). We then have ***Bosons*** and ***Bose-Einstein distribution***. A consequent of Einstein's work is the ***Bose-Einstein Condensation*** at sufficiently low temperatures.

Planck's formula is quite a formula in terms of opening up new areas of research!

Exercises and Think/Learn more…

- Take Planck's formula and work out the long/short wavelengths limits

- Show that Wien's law and Stefan-Boltzmann law follow from Planck's formula

- Carrying out the transformations to frequencies and angular frequencies

- Use a plotting software to plot thermal radiation versus wavelengths for different T

- The limit kBT >> hf gives the classical physics limit. It is related to the idea of each oscillator having the same energy kBT regardless of the frequency. It is called the equipartition of energy in thermal/statistical physics. Self-learn what it is. [It will appear in thermal/statistical physics.]

- There is a pre-factor in Planck's formula related to the dimension (3D,2D,1D) of the system and the EM waves dispersion relation c = f λ. Self-learn what it is. [It will appear again in statistical mechanics and solid state physics (density of states/modes).

- This is a background radiation of 2.7K out there due to the big bang. The work won the 1978 Nobel Prize for Penzias and Wilson. Self-learn how they applied Planck's formula to the work.

- There is something called the "Planck telescope" flying around and measuring the fluctuations in cosmic microwave background (CMB). What is the science?

- Planck's constant h is part of the definition of resistance. How come?

- Planck's constant h is re-defining the Kilogram. What is the story and the science?

- There are many Max Planck's Institute in Germany doing research/education. Germany universities provide free tuition master's degree courses. You may want to look into these opportunities.